Week 3

MATH 34A

TA: Jerry Luo

jerryluo8@math.ucsb.edu

Office Hours: Wednesdays 1:30-2:30PM, South Hall 6431X

Given that there's a midterm this coming Friday based on the material from homeworks 1+2, I decided to compile some of the harder and less done questions from these homeworks. In this packet, there are two question from ~~each of~~ homework 1 and three from homework 2. Here's how today's going to work. As usual, I will devote the first thirty minutes to letting you guys have a crack at these questions. Afterwards, I will present some of these problems. The main difference between today's section and the other sections we've had is that I will not be coming around to help, and there will be no collaboration. The intent of this is to simulate the conditions of an exam (where there will be neither collaboration allowed nor TAs walking you through problems). One thing I want to say is that, **I DO NOT CLAIM THIS WILL BE SIMILAR IN ANY WAY, SHAPE, OR FORM, TO THE ACTUAL MIDTERM.** In particular, I do not have any part in writing the midterm, nor did the professor have any part in the creation/compilation of this worksheet.

1.46 Differentiate $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$.

This is a comp. of functions ~~[illegible]~~ $f(g(x))$

where $g(x) = x + \sqrt{x + \sqrt{x}}$ and $f(y) = \sqrt{y}$.

Using chain rule, we get

$$y' = \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \frac{d}{dx}\left(x + \sqrt{x + \sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \left(1 + \frac{d}{dx}\left(\sqrt{x + \sqrt{x}}\right)\right)$$

Let us find $\frac{d}{dx}\sqrt{x + \sqrt{x}}$ ...

$$= \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \frac{d}{dx}\left(x + \sqrt{x}\right)$$

$$= \frac{1}{2\sqrt{x + \sqrt{x}}} \left(1 + \frac{1}{2\sqrt{x}}\right)$$

Plugging everything in...

$$y' = \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \left[1 + \frac{1}{2\sqrt{x + \sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)\right] \quad //$$

1.5 Find an equation of the tangent line to the curve at the following point: $y = x^2e^{-x}$, $(1, 1/e)$

We use point slope: $y - y_1 = m(x - x_1)$.

$$\Rightarrow y - 1/e = m(x-1) \ldots$$ But what's m?

It's the derivative <u>at</u> $x = 1$.

$$y' = \left(\frac{d}{dx}x^2\right)e^{-x} + x^2\left(\frac{d}{dx}e^{-x}\right)$$

$$= 2xe^{-x} + x^2(-e^{-x})$$

$$= 2xe^{-x} - x^2e^{-x}$$

$$y'(1) = 2(1)e^{-(1)} - (1)^2e^{-(1)}$$

$$= 2e^{-1} - e^{-1} = e^{-1}$$

So, we have $$y - 1/e = e^{-1}(x-1)$$

$$\Rightarrow y = e^{-1}(x-1) + 1/e \quad //$$

2.18 Find the following integrals.

(a) $\int_0^2 e^{8x}dx$

(b) $\int_{-1}^5 5^x dx$

(c) $\int_{-1}^0 (5 \cdot 10^x + x^8)dx$

a) since $\frac{d}{dx} e^{8x} = 8e^{8x}$, we can see that

$\frac{d}{dx} \frac{1}{8} e^{8x} = e^{8x}$.

So, $\int_0^2 e^{8x} dx = \frac{1}{8} e^{8x} \Big|_0^2 = \frac{1}{8} e^{8(2)} - \frac{1}{8} e^{8(0)}$

$= \frac{1}{8} e^{16} - \frac{1}{8}$

b). $\int_{-1}^5 5^x dx = \int_{-1}^5 e^{x\ln 5} dx = \frac{1}{\ln 5} e^{x \ln 5} \Big|_{-1}^5 = \frac{1}{\ln 5} 5^x \Big|_{-1}^5$

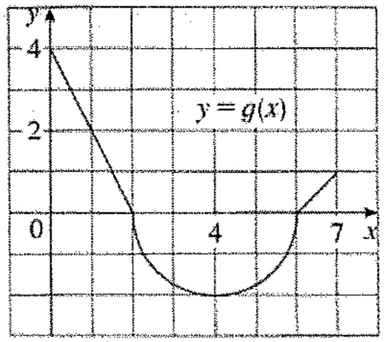
$= \frac{1}{\ln 5} 5^5 - \frac{1}{\ln 5} 5^{-1}$

c) $\int_{-1}^0 5 \cdot 10^x + x^8 dx$. Similar to b), we can show antiderivative of 10^x is $\frac{1}{\ln 10} 10^x$.

So, our integral $= 5 \cdot \frac{1}{\ln 10} 10^x + \frac{x^9}{9} \Big|_{-1}^0 = 5 \cdot \frac{1}{\ln 10} 10^0 + \frac{0^9}{9}$

$- \left(5 \cdot \frac{1}{\ln 10} 10^{-1} + \frac{(-1)^9}{9}\right)$ //.

2.48 Consider the graph of the function $g(x)$:



The graph from $x = 2$ to $x = 6$ is a semicircle. Evaluate the following integrals by interpreting them in terms of areas:

(a) $\int_0^2 g(x)\,dx$

(b) $\int_2^6 g(x)\,dx$

(c) $\int_0^7 g(x)\,dx$

a) Since g is positive from 0 to 2, it is just area under curve from 0 to 2. This is a triangle, with base 2, height 4, which has area 4.

b) $g(x) \le 0$ from 2 to 6, so it is the negative of area below x-axis. It is a semicircle w/ radius 2, so

integral $= -\dfrac{\pi(2)^2}{2}$

c) ~~integral is area~~

$\int_0^7 g(x)\,dx$ is area above x-axis minus area below

$= 4 + \frac{1}{2} - 2\pi$

2.53 Find $\int_0^5 f(x)\,dx$ if

$$f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3. \end{cases}$$

$$\int_0^5 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx$$

$$= \int_0^3 3\,dx + \int_3^5 x\,dx$$

$$= 3x\Big|_0^3 + \frac{x^2}{2}\Big|_3^5$$

$$= 3(3) - 3(0) + \frac{(5)^2}{2} - \frac{(3)^2}{2}$$

$$= 9 + \frac{25}{2} - \frac{9}{2}.$$

$$= 17.$$